Setting a marked price for a property

Key Stage: 3

Strand: Number and Algebra Strand

Learning Unit: Formulae
Linear equations in two unknowns
Inquiry and investigation

Objectives:
(i) To enrich students' experience in applying linear equations in two unknowns and their graphs in real-world scenarios
(ii) To enhance students' abilities in identifying variables required and making assumptions in modelling
(iii) To learn the use of information technology (IT) in the process of mathematical modelling

Prerequisite Knowledge:
(i) Using substitution to find the values of unknowns in the formulae
(ii) Understanding the concept of linear equations in two unknowns and their graphs

Resources Required: Desktop or tablet computers with MS Excel and Internet connection

Background Information:

Real estate pricing is a complex domain that involves various factors influencing the value of a property. Mathematical modelling offers a useful tool to understand and estimate property prices, providing valuable insights to potential buyers, sellers, and real estate professionals. The main goal of the following modelling activities is to enrich students' understanding and practical application of linear equations in two unknowns, alongside their graphical representations.

Rooted in empirical modelling, this set of activities guide students to transform data into a navigational tool for making informed decisions regarding the pricing of a property. With the aid of information technology, students gain the competence to formulate models without being limited by the lack of advanced mathematics and statistics knowledge. As students delve into modelling, they explore numerical

estimations and engage in discussions about variables, assumptions, and limitations of models.

Description of the Activities:

There are three main activities in this resource package:

- Activity 1: To recall prerequisite knowledge (1A) and to plot graphs and formulate models using MS Excel (1B).
- Activity 2: To use data of marked prices of properties alongside their corresponding floor areas to formulate a model.
- Activity 3: To use data of marked prices of properties alongside their corresponding ages of buildings to formulate a model.

Based on Yong et al.'s (2015) framework of the mathematical modelling process, the following table summarises the elements that teachers can discuss with students in the corresponding questions.

Phase	Elements	WS1A	WS1B	WS2	WS3
Define	Define the question of interest	Cover Page			
	Identify variables and parameters	1(a)		1	1
Translate	Identify governing principles	1(a)	2, 4, 6	2	1
	Make simplifying assumptions		9	7	6
	Formulate mathematical model	1(a)	2, 4, 7	4	3
Analyse	Select appropriate math tools & Solve mathematical problem	1(a)	3, 5, 7, 8	3–5	2–4
	Determine or estimate parameters		3, 7	4	3
	Validate solution			6	5
Interpret	Visualise solution	1(b)	3, 7	4	3
	Draw appropriate conclusions & Communicate results	1(a)	5, 8	5, 6	4

Activity 1A (refer to Worksheet 1)

The aim of this activity is to recall students' prerequisite knowledge of using substitution to find the values of unknowns in the formulae (Question 1(a)) and the concept of linear equations in two unknowns and their graphs (Question 1(b)).

Pedagogical recommendations:

1. The teacher can arouse students' interest by discussing the use of formulae in real-world scenarios. Question 1(a) involves a linear equation in two unknowns. The equation can calculate the cost of printing booklets. In Question 1(b), students are tasked to draw the graph of a linear equation in two unknowns. Notice that the slope of the straight line is positive. Therefore, the teacher may discuss the following observation with students.
   - The greater the value of x, the greater the value of y. This will be useful when discussing the relationship between the floor areas and the marked prices of properties in Activity 2.

Suggested solution:

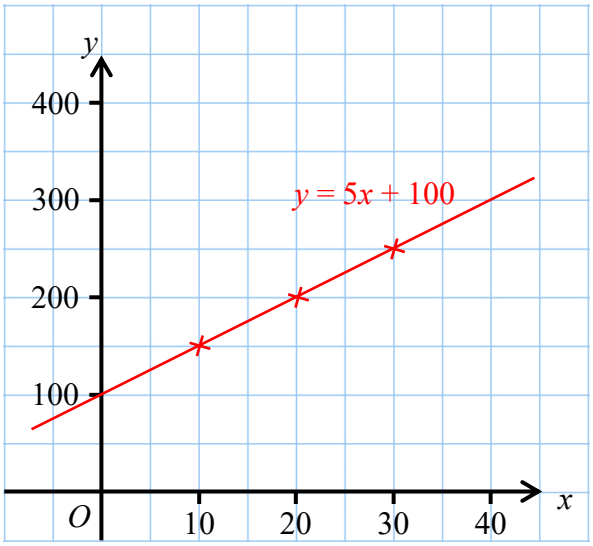
(a) Put $x = 50$ into the equation.

$$y = 5(50) + 100$$
$$= 350$$

$\therefore$ The cost of printing 50 booklets is \$350.

(b) $y = 5x + 100$

x	10	20	30
y	150	200	250



Activity 1B (refer to Worksheet 1)

The aims of this activity are to recall students' skills of using spreadsheets and to introduce the IT skills of using MS Excel in formulating models.

Pedagogical recommendations:

2. Based on the scenario in Question 1(a), the teacher can introduce students to the concept of a model. Specifically, $y = 5x + 100$ can be regarded as a model that describes the relationship between the number of booklets to print and the cost required.

3. The teacher should emphasise that such a calculation is possible because we are given the equation (i.e., the model). In reality, we may not have a model at the very beginning, or we have to formulate a model based on our observation. Therefore, the teacher can introduce students to the IT skills of using MS Excel in formulating models. The teacher can use the step-by-step instructions in Worksheet 1 to guide students in formulating a model that describes the relationship between the number of booklets to print and the time required.

4. Based on the given data, $y = 1.5x + 5$ is a model that describes the relationship between the number of booklets to print and the time required.

5. Question 5 allows students to apply their formulated model to estimate the time required to print certain number of booklets. Through this application, the teacher can emphasise the usefulness of mathematical modelling.

   Suggested solution:

   Put $x = 80$ into $y = 1.5x + 5$.

   $$y = 1.5(80) + 5$$
   $$= 125$$

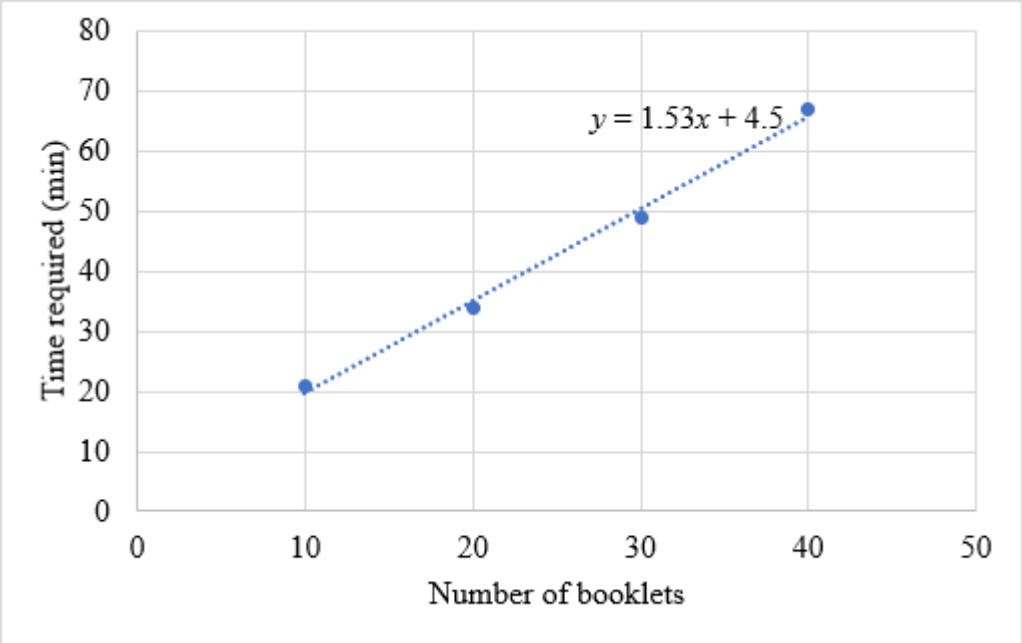
   $\therefore$ The time required to print 80 booklets is estimated at 125 minutes.

6. Notice that the data points provided lie on a straight line. The teacher should emphasise that in reality, it is not always possible to find a straight line that passes through all data points. However, although there are some deviations from a straight line, we can still observe a general trend. For example, the *more* booklets we print, the *more* time is required. For more capable students, the teacher may consider discussing the graphs in 2013 HKDSE Mathematics (Compulsory Part) Paper 2 Question 28 and 2008 HKCEE Mathematics Paper 2 Question 36.

Hence, the teacher can introduce students to the concept of lines of best fit. That is a line that minimises the distance between the line and data points. If students have a strong interest in mathematics and statistics, the teacher may mention the term "regression lines." For junior secondary school students, however, teachers are advised to focus more on the use of IT in the modelling process and the application of modelling outcomes.

7. Question 7 requires students to apply their skills learnt (in Question 3) in plotting a graph and formulating a model based on given data. This serves as a formative assessment for the teacher to evaluate students' mastery of the IT skills required in Activities 2 and 3.

Suggested solution:



∴ The model is $y = 1.53x + 4.5$.

8. Similar to Question 5, Question 8 allows students to apply their formulated model to estimate the time required to print certain number of booklets. Although we cannot find a straight line that passes through all the data points, the line of best fit can help our estimation. Through this application, the teacher can emphasise again the usefulness of mathematical modelling.

   Suggested solution:

   Put $x = 80$ into $y = 1.53x + 4.5$.

   $$y = 1.53(80) + 4.5$$
   $$= 126.9$$

   $\therefore$ The time required to print 80 booklets is estimated at 127 minutes.

9. Toward the end of Activity 1, the teacher can facilitate the discussion on the assumptions and limitations of the formulated model. This can enhance students' abilities in making assumptions and identifying limitations in modelling. The following are some possible discussion outcomes.

   - Assumptions:
     1. Linear relationship: The model assumes that the relationship between the number of booklets and the time required is linear. In reality, this may not always be the case, especially for the printing of a large number of booklets, machine limitations or operator fatigue may slow down the speed.
     2. No other factors: The model assumes that there are no other factors influencing the time required to print booklets other than the number of booklets itself. This might not hold true in practical situations where other variables, such as machine efficiency, operator experience, and varying workload, can affect the time needed.
   - Limitations:
     1. Oversimplification: This model oversimplifies the printing process by considering only one factor (i.e., the number of booklets). Printing can be a complex process influenced by many variables, including paper quality and thickness of booklets, which this model does not consider.
     2. Dependence on a small amount of data: The model is formulated based on only 4 records. Any changes in the printing process, technology, and other relevant factors could make the model less accurate or outdated. A longer period of observation is necessary to collect more data.

Activity 2 (refer to Worksheet 2)

The aim of this activity is to enrich students' modelling experiences by using real data to formulate an empirical model. Students will use marked prices of properties alongside their corresponding floor areas to formulate the model. Worksheet 2 exemplifies how the data can be used and a possible instructional design of guiding students in modelling. To enhance relevance to their daily life, the teacher may ask students to collect data of the marked prices of properties near the school before class. The modelling activities can thus be based on the data collected.

Pedagogical recommendations:

1. The teacher can arouse students' interest by discussing the information that is needed when setting a marked price for a property. This can enhance students' abilities in identifying variables required in modelling. The following are some possible discussion outcomes.
   - Size and layout: Floor area, number of rooms, bathrooms, and kitchens.
   - Condition: New, renovated, or need for refurbishment.
   - Property type: Private housing estate, public housing estate, or village house.
   - Facilities: Swimming pool, gym room, and parking.
   - Location: Proximity to transportation, schools, shopping centres.

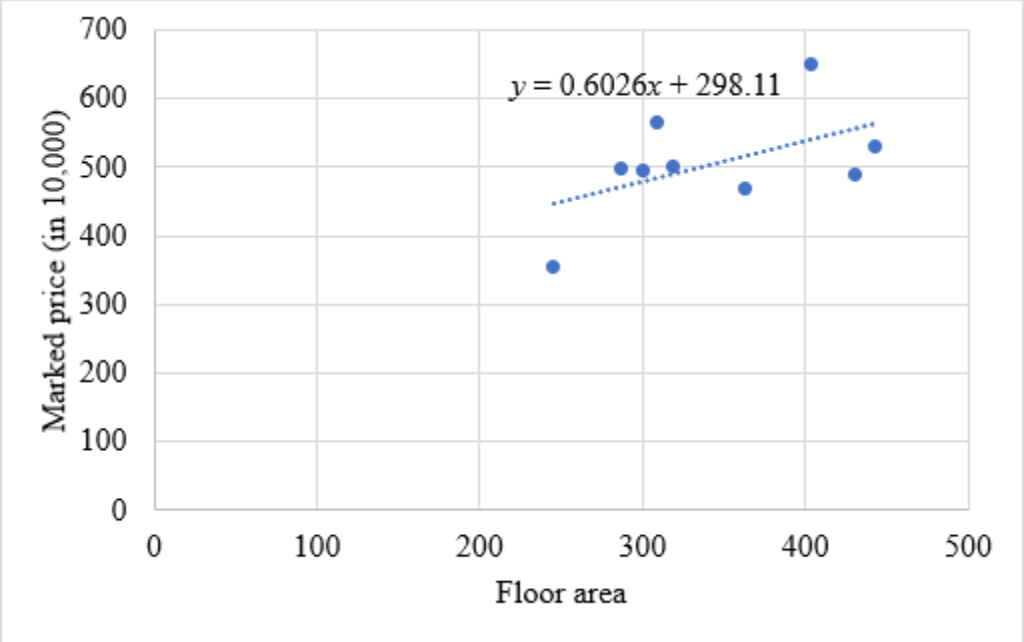
2. The teacher shows some real data of the marked prices of some properties and their corresponding floor areas. Although the ads in the figure of Question 2 are commonly seen in Hong Kong, the teacher should check students' understanding of the values involved. For example, some students may not know how to convert "498 萬" to 4,980,000.

   The teacher can discuss the general relationship between the floor areas and the marked prices of properties. Specifically, the *larger* the floor area, the *higher* the marked price of the property.

3. The teacher then asks students to input the data to MS Excel. To avoid confusion, the teacher can recommend students to input the floor areas and the marked prices in Column A and Column B, respectively.

4. Students use the skills learnt in Activity 1B to plot a graph and formulate a model (Model A) that describes the relationship between the floor areas (x) and the marked price of properties (y). The teacher can point out that the resulting graph matches with the general relationship discussed in Question 2.

Suggested solution:



$\therefore$ The model is $y = 0.6026x + 298.11$.

5. Question 5 allows students to apply their formulated model to suggest a marked price for a property. Through this application, the teacher can emphasise the usefulness of mathematical modelling in real estate pricing.

Suggested solution:

Put $x = 420$ into $y = 0.6026x + 298.11$.

$$y = 0.6026(420) + 298.11$$
$$= 551.202$$

$\therefore$ The marked price for the property can be \$5,510,000.

6. Students are prompted to evaluate the modelling outcome by comparing it with the actual marked price. This is a crucial step that not only aids in evaluating the model but also helps students relate the mathematical world to the real world.

Any discrepancies between the suggested marked prices and the actual marked prices can reveal the needs of refining the model. For example, if the model consistently overestimates or underestimates the marked prices, adjustments to the

model must be made to improve its accuracy.

Suggested solution:

Absolute error

$= \$(580 - 551) \times 10,000$

$= \$290,000$

Percentage error

$= \frac{290,000}{5,800,000} \times 100\%$

$= 5\%$

Our suggested marked price is about 5% lower than the actual marked price. The owner may think that other factors, such as the scenic view and recent renovation of the property, have enhanced its value.

7. Toward the end of Activity 2, the teacher can facilitate the discussion on the assumptions and limitations of the formulated model. This can enhance students' abilities in making assumptions and identifying limitations in modelling. The following are some possible discussion outcomes.
   - Assumptions:
     1. Linear relationship: The model assumes that the relationship between the floor areas and the marked prices of properties is linear. In reality, this may not always be the case. In the real estate market, the price per square feet may grow exponentially as the floor area of properties increases.
     2. No other factors: The model assumes that the marked price is solely influenced by the floor area. This might not hold true in practical situations where other factors, as discussed in Question 1, can affect the pricing.
   - Limitations:
     1. Oversimplification: The model oversimplifies the relationship between the floor areas and the marked prices of properties. In reality, the pricing of a property is influenced by a multitude of factors beyond just floor area, such as location, property condition, market demand, amenities, and economic conditions.
     2. Dependence on a small amount of data: The model is formulated based on the marked prices of only 9 properties, which may not accurately represent the entire range of properties in the market. A more comprehensive dataset is necessary.

Activity 3 (refer to Worksheet 3)

Following Activity 2, the aim of this activity is to further enrich students' modelling experiences by using real data to formulate an empirical model. Students will use marked prices of properties alongside their corresponding ages of buildings to formulate the model. Worksheet 3 exemplifies how the data can be used and a possible instructional design of guiding students in modelling. If students undertake data collection for this activity, they have to find out the year of completion of the buildings included in their dataset. The teacher may recommend students to search for relevant information on the Internet while emphasising the importance of critical thinking on the credibility of their information sources.

Pedagogical recommendations:

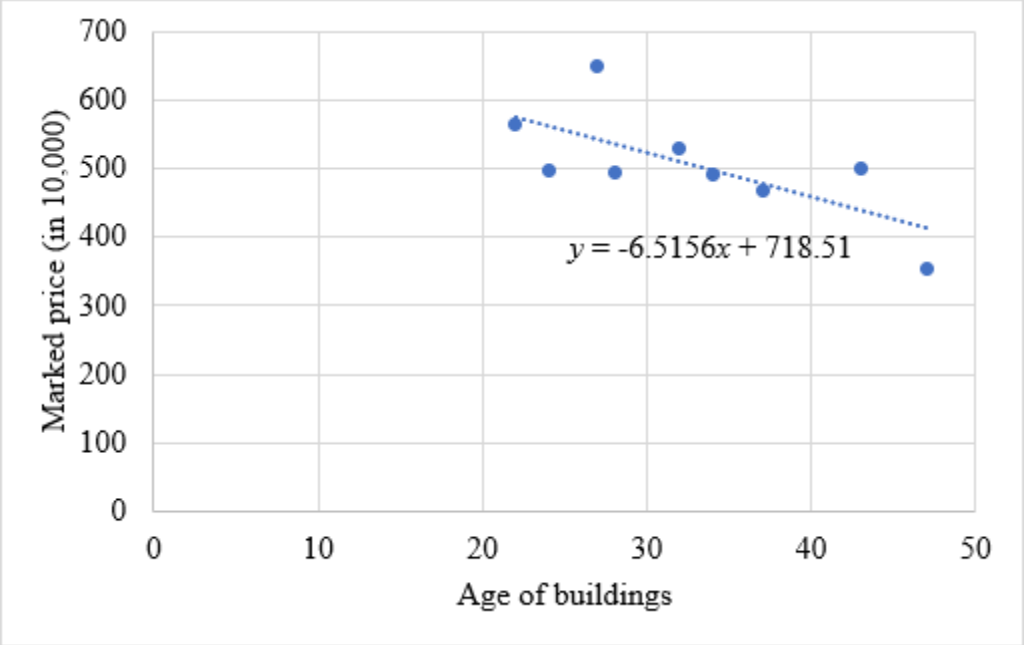
1. The teacher can arouse students' interest by discussing another factor that may affect real estate pricing, namely the age of its building. The teacher shows some real data of the marked prices of some properties and the corresponding years of completion of their buildings.

   The teacher can discuss the general relationship between the ages of buildings and the marked prices of properties. Specifically, the *greater* the age of the building, the *lower* the marked price of the property. This applies the concept of depreciation in the learning unit of "Using percentages."

2. With the experiences in Activity 2, students should be able to input the data to MS Excel. However, the teacher should check whether they are able to obtain the ages of buildings from their years of completion. To avoid confusion, the teacher can recommend students to input the ages of buildings and the marked prices in Column A and Column B, respectively.

3. Students use the skills learnt in Activity 1B to plot a graph and formulate a model (Model B) that describes the relationship between the ages of buildings (x) and the marked prices of properties (y). The teacher can point out that the resulting graph matches with the general relationship discussed in Question 1.

Suggested solution:



∴ The model is $y = -6.5156x + 718.51$.

4. Question 4 allows students to apply their formulated model to suggest a marked price for a property.

   Suggested solution:

   The age of the building

   $= 2023 - 1985$

   $= 38$

   Put $x = 38$ into $y = -6.5156x + 718.51$.

   $y = -6.5156(38) + 718.51$

   $= 470.9172$

   ∴ The marked price for the property can be \$4,710,000.

5. With the suggested marked prices of Model A and Model B, students are given the opportunity to compare different models. For Model A, there is a difference of \$290,000. But for Model B, there is a difference of \$1,090,000. In this case, the suggested marked price of Model A is closer to the actual marked price than that of Model B. The following is a possible implication.

   - Compared with the floor area, the age of a building may be less influential on the marked price of a property.

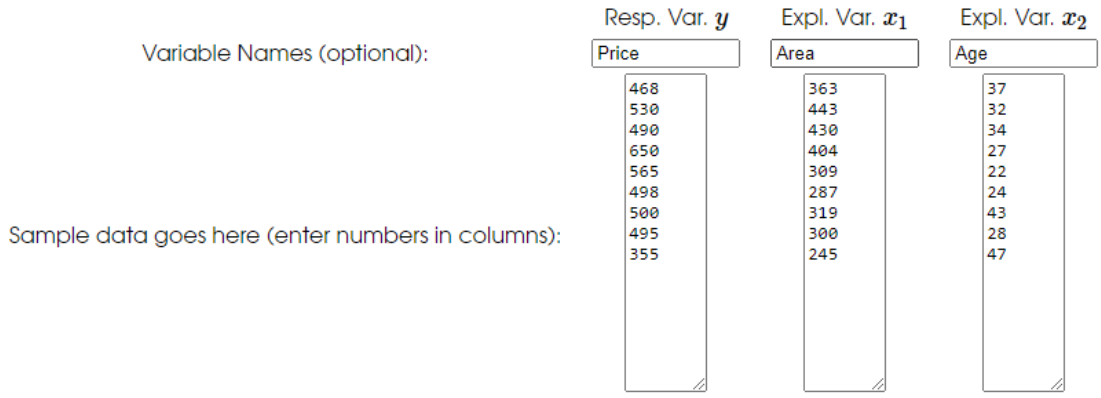
6. Similar to Activity 2, the teacher can facilitate the discussion on the assumptions and limitations of the formulated model. With the experiences in Activity 2, students should be more able to identify the assumptions made and limitations in modelling. The following are some possible discussion outcomes which are similar to those in Question 7 of Activity 2.
   - Assumptions:
     1. Linear relationship: The model assumes that the relationship between the ages of buildings and the marked price of properties is linear.
     2. No other factors: The model assumes that the marked price is solely influenced by the ages of buildings. However, at least one important factor, namely floor areas, has not been considered.
   - Limitations:
     1. Oversimplification: The model oversimplifies the relationship between the ages of buildings and the marked prices of properties.
     2. Dependence on a small amount of data: The model is formulated based on the marked prices of only 9 properties, which may not accurately represent the entire range of properties in the market.

Concluding remarks:

In Activities 2 and 3, students used MS Excel to formulate the lines of best fit based on some real data. These lines are useful to express the relationship of different data points. However, both Activities 2 and 3 only consider one factor (the floor area; the age of a building) when formulating the model.

To construct a model involving two or more factors, the teacher can consider introducing students to the use of free online statistical applications, such as:
https://stats.blue/Stats_Suite/multiple_linear_regression_calculator.html
https://www.socscistatistics.com/tests/multipleregression/default.aspx

As an enrichment activity, students may use the data of the above activities to formulate a model.



However, it is worth noting that a small amount of data is involved in the above activities. Besides, there are many other factors that affect how property owners set their marked price. Therefore, we have to collect more data as well as continue to refine our model by considering more factors.

Reference:

Yong, D., Levy, R., & Lape, N. (2015). Why no difference? A controlled flipped classroom study for an introductory differential equations course. *PRIMUS, 25*(9–10), 907–921.

Suggested lesson plans and teaching flow

Teaching time: 70 minutes or a double lesson

Time (mins)	Objectives	Teaching activities and processes	Resources/ remarks
10	• To arouse students' interest • To recall prerequisite knowledge	1. The teacher arouses students' interest by discussing the real-world scenario. 2. The teacher uses an example as a warm-up exercise to recall students' knowledge of using substitution to find the values of unknowns in the formulae. 3. The teacher uses an example as a warm-up exercise to recall students' concept of linear equations in two unknowns and their graphs.	WS cover page WS1A Q1
25	• To introduce the concept of a model • To recall the IT skills of using spreadsheets • To introduce the IT skills of using MS Excel in formulating models • To apply modelling outcomes	1. The teacher introduces students to the concept of a model. For example, the equation $y = 5x + 100$ can be regarded as a model that describes the relationship between the number of booklets to print and the cost required. 2. The teacher asks students to input data to MS Excel to recall their skills of using spreadsheets. 3. The teacher introduces students to the IT skills of using MS Excel in formulating models. 4. The teacher asks students to apply their modelling outcomes in estimation.	WS1B Q2 WS1B Q3–4 WS1B Q5

Time (mins)	Objectives	Teaching activities and processes	Resources/ remarks
	• To introduce the concept of lines of best fit	5. The teacher emphasises that in reality, it is not always possible to find a straight line that passes through all data points. However, we can still observe a general trend. The following two questions can be showcased. ➢ 2013 HKDSE Math Paper 2 Q28 ➢ 2008 HKCEE Math Paper 2 Q36 6. The teacher thus introduces the concept of lines of best fit.	WS1B Q6
	• To reinforce the IT skills of using MS Excel in formulating models	7. The teacher asks students to apply their stills learnt in plotting a graph and formulating a model based on given data. 8. The teacher evaluates students' mastery of the IT skills.	WS1B Q7
	• To apply modelling outcomes	9. The teacher asks students to apply their modelling outcomes in estimation.	WS1B Q8
	• To enhance abilities in making assumptions and identify limitations in modelling	10. Students discuss in groups the assumptions and limitations of the formulated model. The teacher may ask the following questions to stimulate thinking: Is the relationship always linear? Are there other factors that we have not considered? Are the data adequate to formulate the model?	WS1B Q9

Time (mins)	Objectives	Teaching activities and processes	Resources/ remarks
20	• To enhance abilities in identifying variables	1. Students discuss in groups the information needed when setting a marked price for a property. The teacher may ask students to consider the following aspects to facilitate discussion: Size and layout, condition, property type, facilities, and location.	WS2 Q1
	• To formulate an empirical model using real data	2. By showing some real data, the teacher discusses the general relationship between the floor areas and the marked prices of properties. 3. The teacher asks students to input the data to MS Excel and formulate the model (Model A).	WS2 Q2–4
	• To apply modelling outcomes	4. The teacher asks students to apply their modelling outcomes to propose a suggested marked price.	WS2 Q5
	• To evaluate the model	5. The teacher asks students to evaluate the modelling outcome by comparing it with the real data. 6. Students discuss in groups the possible reasons of the difference between the actual marked price and the suggestion based on the model. The teacher may ask the following question to stimulate thinking: Why does the owner believe that the property merits a higher price?	WS2 Q6

Time (mins)	Objectives	Teaching activities and processes	Resources/ remarks
	• To enhance abilities in making assumptions and identify limitations in modelling	7. Students discuss in groups the assumptions and limitations of the formulated model. The teacher may ask the following questions to stimulate thinking: Is the relationship always linear? Are there other factors that we have not considered? Are the data adequate to formulate the model?	WS2 Q7
10	• To formulate an empirical model using real data	1. By showing some real data, the teacher discusses the general relationship between the ages of buildings and the marked prices of properties. 2. The teacher asks students to input the data to MS Excel and formulate the model (Model B).	WS3 Q1–3
	• To apply modelling outcomes	3. The teacher asks students to apply their modelling outcomes to provide suggestion.	WS3 Q4
	• To evaluate the model	4. The teacher asks students to evaluate the modelling outcome by comparing it with the real data. 5. The teacher then asks students to compare different models. Students discuss in groups the implication of their observation. The teacher may ask the following questions to stimulate thinking: Which model can suggest a marked price which is closer to the actual marked price? What factor is more influential on the marked price?	WS3 Q5

Time (mins)	Objectives	Teaching activities and processes	Resources/ remarks
	• To enhance abilities in making assumptions and identify limitations in modelling	6. Students discuss in groups the assumptions and limitations of the formulated model. The teacher may ask the following questions to stimulate thinking: Is the relationship always linear? Are there other factors that we have not considered? Are the data adequate to formulate the model?	WS3 Q6
5	• To conclude the activity	1. The teacher recalls the concept of a model and the use of MS Excel in formulating models. 2. The teacher emphasises the usefulness of mathematical modelling in real estate pricing as well as other disciplines. 3. The teacher acknowledges the needs of making assumptions when formulating the models, and thus the limitations of the models. 4. The teacher suggests students' further investigation of constructing a model involving two or more factors using free online statistical applications.	